# PET ENGINEERING COLLEGE

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## DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

## UNIT - I

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|---|---|
| **CLASS** | **: S5 ECE** |
| **SUBJECT CODE** | **: EC3551** |
| **SUBJECT NAME** | **: TRANSMISSION LINES AND RF SYSTEMS** |
| **REGULATION** | **: 2021** |

02.08.23

# Unit-1

## Transmission line theory

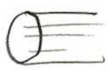
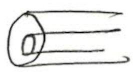
### General theory of transmission lines

Transmission line: Transmission of electrical signal from source to receiver.

Lumped parameters : R, L, C discrete point

Distributed parameter : R, L, C

Types of transmission line:

- Open wire line =
- cables 
- co-axial line 

Primary constant

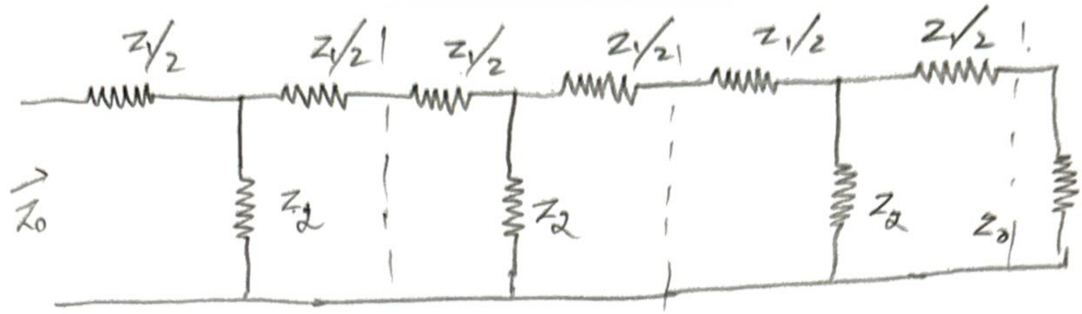
- Resistance, R ($\Omega$)
- Inductance, L (H)
- Capacitance, C (F)
- Conductance G (℧)

$Z \Rightarrow$ impedance $\Rightarrow R + j\omega L$

$Y \Rightarrow$ admittance $\Rightarrow G + j\omega C$

Transmission line Theory:

line of cascaded T section




(i) Characteristic impedance :

$$Z_{0T} = \sqrt{Z_1 Z_2\left(1+\frac{Z_1}{4Z_2}\right)} \rightarrow (1)$$

$$Z_1 = Z\Delta x$$

$$Z_2 = \frac{1}{y\Delta x}$$

Substitute in eqn (1)

$$Z_{0T} = \sqrt{\frac{Z\Delta x}{y\Delta x}\left(1+\frac{\frac{Z\Delta x}{4}}{\frac{1}{y\Delta x}}\right)} = \sqrt{\frac{Z}{y}\left(1+\frac{yZ(\Delta x)^2}{4}\right)}$$

Assume $\Delta x = 0$

$$\boxed{Z_{0T} = \sqrt{\frac{Z}{y}}}$$

(ii) Propagation constant:

By Binomial theorem, $(a+b)^{1/2} = a + \frac{1}{2}b$

$$\sqrt{\frac{Z_1}{Z_2}\left(1+\frac{Z_1}{4Z_2}\right)} = \sqrt{\frac{Z_1}{Z_2}}\left(1+\frac{Z_1}{4Z_2}\right)^{1/2}$$

$$= \boxed{\sqrt{\frac{Z_1}{Z_2}}\left(1+\frac{1}{2}\right.}\left.\times\frac{Z_1}{4Z_2}\right) \rightarrow a$$

W.K.T.

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2}\left(1+\frac{Z_1}{4Z_2}\right)} \longrightarrow (b)$$

Sub (a) in (b)

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2}} + \frac{1}{8} \times \sqrt{\frac{Z_1}{Z_2}} \times \frac{Z_1}{Z_2}$$

$$= 1 + \frac{\sqrt{\frac{Z_1}{Z_2}}}{1!} + \frac{\left(\sqrt{\frac{Z_1}{Z_2}}\right)^2}{2!} + \frac{\left(\sqrt{\frac{Z_1}{Z_2}}\right)^3}{3!}$$

W.K.T

$$e^{\gamma} = 1 + \frac{\gamma}{1!} + \frac{\gamma^2}{2!} + \frac{\gamma^3}{3!} \rightarrow (d)$$

Compare (c) & (d)

$$\gamma = \sqrt{\frac{Z_1}{Z_2}}$$

By extended line $\Delta x$

$$Z_1 = Z\Delta x \qquad Z_2 = \frac{1}{y\Delta x}$$

$$\gamma = \sqrt{\frac{Z\Delta x}{\frac{1}{y\Delta x}}}$$

$$\gamma = \sqrt{yZ(\Delta x)^2}$$

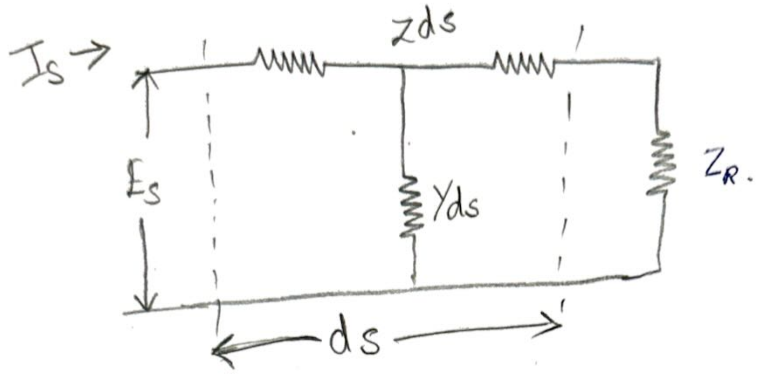
if $\Delta x = 1$

$$\boxed{\gamma = \sqrt{Zy}}$$

# The transmission line General solution.

→ infinitesimal T-section

→ length (ds), current (I)




Voltage drop. $dE = I.z\,ds$

$$\frac{dE}{ds} = I.z \longrightarrow (1)$$

Current, $dI = E.Y\,ds$

$$\frac{dI}{ds} = E.Y \longrightarrow (2)$$

differentiate (1) & (2)

$$\frac{d^2E}{ds^2} = z\,\frac{dI}{ds} = z.EY \longrightarrow (3)$$

$$\frac{d^2I}{ds^2} = Y.\frac{dE}{ds} = Y.IZ \longrightarrow (4)$$

$$\frac{d^2}{ds^2} = m^2$$

$$m^2E = ZEY.$$

$$m^2E - ZEY = 0$$

$$m^2 - ZY = 0$$

$$m^2 = ZY$$

$$m = \sqrt{ZY}$$

Solution of differential equation

$$E = Ae^{ms} + Be^{-ms}$$

$$I = Ce^{ms} + De^{-ms}$$

Sub m

$$E = Ae^{\sqrt{ZY}s} + Be^{-\sqrt{zy}\,s} \longrightarrow (5)$$

$$I = Ce^{\sqrt{ZY}s} + De^{-\sqrt{ZY}\,s} \longrightarrow (6)$$

Condition: 1

$S = 0$, $E = E_R$, $I = I_R$ $\qquad e^0 = 1$

$$E_R = A + B \longrightarrow (7)$$

$$I_R = C + D \longrightarrow (8)$$

Differentiate (5) & (6) with respect to 's'.

$$\frac{dE}{ds} = A\cdot\sqrt{zy}\,e^{\sqrt{zy}\,s} - B\sqrt{zy}\,e^{-\sqrt{zy}s}$$

$$\frac{dI}{ds} = C\sqrt{zy}\,e^{\sqrt{zy}\,s} - D\sqrt{zy}\,e^{-\sqrt{zy}\,s}$$

W.K.T

$$\frac{dE}{ds} = IZ, \quad \frac{dI}{ds} = EY$$

$$IZ = A\sqrt{zy}\,e^{\sqrt{zy}s} - B\sqrt{zy}\,e^{-\sqrt{zy}s}$$

$$I = A\sqrt{y/z}\, e^{\sqrt{zy}\,s} - B\sqrt{y/z}\, e^{-\sqrt{zy}\,s} \rightarrow (9)$$

$$EY = C\sqrt{zy}\, e^{\sqrt{zy}\,s} - D\sqrt{zy}\, e^{-\sqrt{zy}\,s}$$

$$E = C\sqrt{\frac{z}{y}}\, e^{\sqrt{zy}\,s} - D\sqrt{\frac{z}{y}}\, e^{-\sqrt{zy}\,s} \rightarrow 10$$

Sub $s=0$, $E = E_r$, $I = I_R$ in eqn (9) & 10

$$I_R = A\sqrt{y/z} - B\sqrt{y/z}$$

$$E_R = C\sqrt{z/y} - D\sqrt{z/y}$$

$$\sqrt{y/z}\,(A-B) = I_R$$

$$A - B = \sqrt{z/y}\, I_R \rightarrow (11)$$

$$\sqrt{z/y}\,(C-D) = E_r$$

$$C - D = \sqrt{y/z}\, E_r \rightarrow (12)$$

Adding (7) & (11)

$$A + B = E_r$$

$$A - B = \sqrt{z/y}\, I_R$$

$$2A = E_R + \sqrt{z/y}\, I_R$$

$$A = \frac{E_R}{2} + \sqrt{Z/Y}\,\frac{I_R}{2}.$$

[(7) eqn]

$$E_R = A + B$$

$$E_R = \frac{E_R}{2} + \sqrt{\frac{Z}{Y}}\,\frac{I_R}{2} + B$$

$\sqrt{Z/Y} = Z_0$

$$= \frac{E_R}{2} + \frac{Z_0 I_R}{2} + B$$

$\sqrt{Y/Z} = 1/Z_0$

$$E_R = \frac{E_R + Z_0 I_R + 2B}{2}$$

$$2E_R = E_R + Z_0 I_R + 2B$$

$$E_R = Z_0 I_R + 2B$$

$$E_R - Z_0 I_R = 2B$$

$$B = \frac{E_R - Z_0 I_R}{2}.$$

Adding (8) & (12)

$$I_R = C + D$$

$$\sqrt{Y/Z}\,E_R = C - D$$

$$I_R + \sqrt{Y/Z}\,E_R = 2C$$

$$C = \frac{I_R}{2} + \sqrt{Y/Z}\,\frac{E_R}{2}$$

$$C = \frac{I_R}{2} + \frac{1}{Z_0}\frac{E_R}{2}.$$

Substitute $C$ in eqn (8)

$$I_R = C + D$$

$$I_R = \frac{I_R}{2} + \frac{E_R}{2Z_0} + D$$

$$I_R = \cancel{\frac{I_R + E_R + 2D}{}} \frac{I_R Z_0 + E_R + D2Z_0}{2Z_0}$$

$$2 I_R Z_0 = I_R Z_0 + E_R + 2DZ_0$$

$$I_R Z_0 = E_R + 2DZ_0$$

$$I_R Z_0 - E_R = 2DZ_0$$

$$D = \frac{I_R Z_0 - E_R}{2Z_0}$$

$$D = \frac{I_R}{2} - \frac{E_R}{2Z_0}$$

Take $\frac{E_R}{2}$ out in A & B

$$A = \frac{E_R}{2}\left[1 + \frac{Z_0 I_R}{2} \times \frac{2}{E_R}\right]$$

$$= \frac{E_R}{2}\left(1 + \frac{I_R Z_0}{E_R}\right) \qquad \left[Z_R = \frac{E_R}{I_E}\right]$$

$$A = \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right)$$

$$B = \frac{E_R}{2}\left(1 - \frac{I_R Z_0}{E_R}\right)$$

$$B = \frac{E_R}{2}\left(1 - \frac{Z_0}{Z_R}\right)$$

Taking $\frac{I_R}{2}$ out in C & D

$$C = \frac{I_R}{2}\left(1 + \frac{E_R}{Z_0 I_R}\right)$$

$$= \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_0}\right)$$

$$D = \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_0}\right)$$

$$E = Ae^{\sqrt{zy}\,s} + Be^{-\sqrt{zy}\,s}$$

$$= \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right)e^{\sqrt{zy}\,s} + \frac{E_R}{2}\left(1 - \frac{Z_0}{Z_R}\right)e^{-\sqrt{zy}\,s}$$

$$I = Ce^{\sqrt{zy}\,s} + De^{-\sqrt{zy}\,s}$$

$$= \frac{I_R}{2}\left(1 + \frac{E_R}{Z_0 I_R}\right)e^{\sqrt{zy}\,s} + \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_0}\right)e^{-\sqrt{zy}\,s}$$

$$= \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_0}\right)e^{\sqrt{zy}\,s} + \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_0}\right)e^{-\sqrt{zy}\,s}$$

04.08.23

# Physical Significance of Transmission line:-

Infinite line

$$E = \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right)e^{\sqrt{zy}s} + \frac{E_R}{2}\left(1 - \frac{Z_0}{Z_R}\right)e^{-\sqrt{zy}s}$$

$$I = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_0}\right)e^{\sqrt{zy}s} + \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_0}\right)e^{-\sqrt{zy}s}$$

$$E = \frac{E_R}{2}e^{\sqrt{zy}s} + \frac{E_R}{2}\frac{Z_0}{Z_R}e^{\sqrt{zy}s} + \frac{E_R}{2}e^{-\sqrt{zy}s} - \frac{E_R}{2}\frac{Z_0}{Z_R}e^{-\sqrt{zy}s}$$

$$= \frac{E_R}{2}\left(e^{\sqrt{zy}s} + e^{-\sqrt{zy}s}\right) + \frac{E_R}{2}\frac{Z_0}{Z_r}\left(e^{\sqrt{zy}s} - e^{-\sqrt{zy}s}\right)$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\gamma = \sqrt{zy}$$

$$E = E_R \cosh \gamma s + \frac{E_R Z_0}{Z_R}\sinh \gamma s$$

$$\boxed{E = E_R\left[\cosh \gamma s + \frac{Z_0}{Z_R}\sinh \gamma s\right]}$$

$$I = \frac{I_R}{2}e^{\sqrt{zy}s} + \frac{I_R Z_R}{2Z_0}e^{\sqrt{zy}s} + \frac{I_R}{2}e^{-\sqrt{zy}s} - \frac{I_R Z_R}{2Z_0}e^{-\sqrt{zy}s}$$

$$I = \frac{I_R}{2}\left[e^{\sqrt{zy}s} + e^{-\sqrt{zy}s}\right] +$$

$$\frac{I_R Z_R}{2Z_0}\left[e^{\sqrt{zy}s} - e^{-\sqrt{zy}s}\right]$$

$$= \frac{I_R}{2} 2\cosh\gamma s + \frac{I_R Z_R}{2Z_0} 2\sinh\gamma s$$

$$\boxed{I = I_R\left[\cosh\gamma s + \frac{Z_R}{Z_0}\sinh\gamma s\right]}$$

At point $s = l$, $E = E_S$, $I = I_S$

$$E_S = E_R\left[\cosh\gamma l + \frac{Z_0}{Z_R}\sinh\gamma l\right]$$

$$I_S = I_R\left[\cosh\gamma l + \frac{Z_R}{Z_0}\sinh\gamma l\right]$$

$$Z_S = \frac{E_S}{I_S}$$

Case (i) if $Z_R = Z_0$

$$E_S = E_R\left[\cosh\gamma l + \frac{Z_0}{Z_0}\sinh\gamma l\right]$$

$$I_S = I_R\left[\cosh\gamma l + \sinh\gamma l\right]$$

$$Z_S = \frac{E_S}{I_S} = \frac{E_R}{I_R}\left[\frac{\cosh\gamma l + \sinh\gamma l}{\cosh\gamma l + \sinh\gamma l}\right]$$

$$Z_S = Z_R$$

if $Z_S = Z_R$

$Z_0 = Z_S$

case ii) if $Z_R \neq Z_0$

$$Z_S = \frac{E_S}{I_S} = \frac{E_R\left[\cosh\gamma l + \frac{Z_0}{Z_R}\sinh\gamma l\right]}{I_R\left[\cosh\gamma l + \frac{Z_R}{Z_0}\sinh\gamma l\right]}$$

$$= \frac{Z_R\left[\frac{Z_R\cosh\gamma l + Z_0\sinh\gamma l}{Z_R}\right]}{\left[\frac{Z_0\cosh\gamma l + Z_R\sinh\gamma l}{Z_0}\right]}$$

$$Z_S = \frac{Z_0\left(Z_R\cosh\gamma l + Z_0\sinh\gamma l\right)}{Z_0\cosh\gamma l + Z_R\sinh\gamma l}$$

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Wavelength and velocity of propagation:- infinite line.

Wavelength:

$\lambda \propto 1/\beta$ (phase constant)

wavelength is inversely proportional to phase constant

$$\lambda = \frac{2\pi}{\beta}$$

Velocity:

velocity is defined as the ratio of angular frequency($\omega$) to phase constant ($\beta$)

$$\boxed{V = \omega/\beta}$$

$$\gamma = \sqrt{zy}$$

$$Z = R + j\omega L \qquad\qquad \gamma = \alpha + j\beta \rightarrow (1)$$

$$Y = G + j\omega C$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{RG + j\omega RC + j\omega LG - \omega^2 LC}$$

$$\gamma = \sqrt{RG - \omega^2 LC + j\omega(RC + LG)} \rightarrow (2)$$

Equate (1) & (2)

$$\alpha + j\beta = \sqrt{RG - \omega^2 LC + j\omega(RC + LG)}$$

$$(\alpha + j\beta)^2 = RG - \omega^2 LC + j\omega(RC + LG)$$

$$\alpha^2 - \beta^2 + 2\alpha\beta j = RG - \omega^2 LC + j\omega(RC + LG)$$

Equating real part

$$\alpha^2 - \beta^2 = RG - \omega^2 LC \longrightarrow (3)$$

Imaginary part.

$$2\alpha\beta = (RC + LG)\omega$$

$$\alpha = \frac{\omega(RC + LG)}{2\beta}$$

Sub $\alpha$ in eqn (3)

$$\frac{\omega^2(RC+LG)^2}{4\beta^2} - \beta^2 = RG - \omega^2 LC$$

$$\frac{\omega^2(RC+LG)^2 - 4\beta^4}{4\beta^2} = RG - \omega^2 LC$$

If $R = G = 0$

$$-\beta^2 = -\omega^2 LC$$

$$\beta = \sqrt{\omega^2 LC}$$

$$\beta = \omega\sqrt{LC}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} \; m$$

$$V = \omega/\beta = \frac{\omega}{\omega\sqrt{LC}}$$

$$V = \frac{1}{\sqrt{LC}} \; ms^{-1}$$

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**Waveform distortion:**

There are two types,

i) frequency distortion

ii) phase distortion

## Frequency distortion:

When the signal has more than one frequency, signals are not attenuated then it is called frequency distortion

## Phase distortion: (delay)

When the signal has more than one frequency the signals are not transmitted equally, a delay is introduced between these signal which leads to phase distortion.

07.08.23 **Distortionless lines:**

⇒ If a line is said to be distortionless lines, the $\alpha$ and velocity (v) is independent of frequency,

⇒ The condition of distortionless line

$$LG = RC$$

$$\gamma = \sqrt{ZY}$$

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{RG + j\omega RC + j\omega LG + j^2\omega^2 LC}$$

$$= \sqrt{RG + j\omega RC + j\omega RC + j^2\omega^2 LC}$$

$$= \sqrt{RG + j^2\omega^2 RC + 2j\omega RC}$$

$$RG = \left(\sqrt{RG}\right)^2$$

$$j^2\omega^2 LC = \left(\sqrt{j^2\omega^2 LC}\right)^2$$

$$= \sqrt{\left(\sqrt{RG}\right)^2 + \left(\sqrt{j^2\omega^2 LC}\right)^2 + 2j\omega RC}$$

$$= \sqrt{\left(\sqrt{RG}\right)^2 + \left(\sqrt{j^2\omega^2 LC}\right)^2 + 2j\omega \sqrt{RC}\sqrt{RC}}$$

$$= \sqrt{\left(\sqrt{RG}\right)^2 + \left(\sqrt{j^2\omega^2 LC}\right)^2 + 2j\omega \sqrt{RC}\sqrt{LG}}$$

$$= \sqrt{\left(\sqrt{RG}\right)^2 + \left(\sqrt{j^2\omega^2 LC}\right)^2 + 2j\omega \sqrt{RG}\sqrt{LC}}$$

$$= \sqrt{\left(\sqrt{RG} + \sqrt{j^2\omega^2 LC}\right)^2}$$

$$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$$

$$\gamma = \alpha + j\beta$$

$\alpha = \sqrt{RG}$ $\beta = \omega\sqrt{LC}$

$$V = \omega/\beta = \frac{\omega}{\omega\sqrt{LC}}$$

$$= 1/\sqrt{LC}$$

Here $\alpha$ & $V$ are independent of frequency.

## Loading of lines:

⇒ In order to achieve distortionless lines, the ratio of $L/C$ should be increased.

⇒ It is done by increasing inductance (L) in transmission line by inserting inductance (L) in series. This is called loaded lines. This loading coil is called lumped inductors.

Telephone Cable: (distorted line)

* twisted pair cable (insulated)
* audio frequency $\boxed{L = G = 0}$

$$\gamma = \sqrt{ZY}$$

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\gamma = \sqrt{(R+0)(0+j\omega C)}$$

$$\gamma = \sqrt{j\omega RC}$$

$$(1+j)^2 = 1 + j^2 + 2j$$
$$= 1 - 1 + 2j$$

$$\gamma = \sqrt{\frac{2 \times j\omega RC}{2}}$$

$$(1+j)^2 = 2j$$
$$1+j = \sqrt{2j}$$

$$= (1+j)\sqrt{\frac{\omega RC}{2}}$$

$$\alpha + j\beta = \sqrt{\frac{\omega RC}{2}} + j\sqrt{\frac{\omega RC}{2}}$$

$$\alpha = \sqrt{\frac{\omega RC}{2}} \qquad \beta = \sqrt{\frac{\omega RC}{2}}$$

$$V = \omega/\beta = \frac{\omega\sqrt{2}}{\sqrt{\omega RC}}$$

$$V = \sqrt{\frac{2\omega}{RC}}$$

$\alpha$ & $V$ &

.23 Types of loading:

i) Inductance loading

ii) Continuous loading

iii) Patch loading

Inductance loading:

In this, inductance (L) is increased by introduction of loading coils at uniform interval.

Continuous loading

Iron or other magnetic material is wounded on transmission line to increase permeability of surrounding to increase inductance.

Patch loading.

It employs section of loaded cable which is seperated by unloaded cable.

Inductance loading of telephone cable.

In order to make telephone cable as distortionless cable, it is necessary to increase the inductance (L) and keep $G = 0$. Increasing the inductance is called inductance loading.

$$\gamma = \sqrt{ZY}$$

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

$$Z = |Z| \cdot \angle Z$$

$$= \sqrt{R^2 + \omega^2 L^2} \cdot \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$= \sqrt{R^2 + \omega^2 L^2}\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{R}{\omega L}\right)\right)$$

$$Y = |Y| \cdot \angle Y$$

$$= \sqrt{\omega^2 C^2} \tan^{-1}\left(\frac{\omega C}{0}\right)$$

$$= \omega C \tan^{-1}(\infty)$$

1.08.23

$$= \pi/2 \ \omega C$$

$$Z = \sqrt{R^2 + \omega^2 L^2}\left(\pi/2 - \tan^{-1}\left(\frac{R}{\omega L}\right)\right) \rightarrow (1)$$

$$Y = \pi/2 \ \omega C \longrightarrow (2)$$

$$\gamma = \sqrt{ZY}$$

$$= \sqrt{\sqrt{\omega^2 L^2}\left(1+\frac{R^2}{\omega^2L^2}\right)^{1/2} \angle \pi/2 - \tan^{-1}\left(R/\omega L\right)}$$

$$\sqrt{\pi/2\omega C}$$

$$= \sqrt{\omega^2 LC\left(1+\frac{R^2}{\omega^2L^2}\right)^{1/2} \angle \pi/2 - \tan^{-1}\left(R/\omega L\right)}$$

$$= \omega\sqrt{LC}\left[\left(1+\frac{R^2}{\omega^2L^2}\right)\left(\angle \pi - \tan^{-1}\left(R/\omega L\right)\right)\right]^{1/2}$$

$\frac{R^2}{\omega^2L^2}$ is small

$$\angle a^{1/2} = \frac{\angle a}{2}$$

$$= \omega\sqrt{LC}\ \frac{\angle \pi - \tan^{-1}(R/\omega L)}{2}$$

$$\gamma = \omega\sqrt{LC}\ \angle\left[\pi/2 - 1/2 \tan^{-1}\left(\frac{R}{\omega C}\right)\right] \rightarrow (3)$$

$$\gamma = |\gamma| \angle \gamma \quad ; \quad e^{j\theta} = \cos\theta + j\sin\theta$$

$$\gamma = |\gamma| e^{j\theta} \longrightarrow (4)$$

$$\theta = \pi/2 - \tan^{-1}\left(R/\omega L\right) \rightarrow (5)$$

Taking cos on both sides

$$\cos\theta = \cos\left(\pi/2 - \frac{1}{2}\tan^{-1}(R/\omega L)\right)$$

$$= \sin\left(\frac{1}{2}\tan^{-1}\left(\frac{R}{\omega L}\right)\right)$$

$$= \frac{1}{2}\tan^{-1}(R/\omega L)$$

$$\cos\theta = \frac{1}{2}\frac{R}{\omega L}.$$

$\theta$ is small

$\cos\theta = 1$

$\sin\theta = \tan\theta = \theta$

$\tan^{-1}\theta = \theta$

$\cos 90^\circ - \theta = \sin\theta$

$\sin 90^\circ - \theta = \cos\theta$

Taking sine of both sides (5)

$$\sin\theta = \sin\left(\pi/2 - \frac{1}{2}\tan^{-1}(R/\omega L)\right)$$

$$= \cos\left(\frac{1}{2}\tan^{-1}(R/\omega L)\right)$$

$$\sin\theta = 1$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{j\theta} = \frac{1}{2}\frac{R}{\omega L} + j$$

$$\gamma = |\gamma|\angle\gamma = \omega\sqrt{LC}\left(\frac{R}{2\omega L} + j\right)$$

$$\gamma = \omega\sqrt{LC}\left(\frac{R}{2\omega L} + j\right)$$

$$\gamma = \frac{\omega\sqrt{LC}\,R}{2\omega L} + \omega\sqrt{LC}\,j$$

$$\alpha + j\beta = \frac{R}{2}\sqrt{\frac{C}{L}} + j\omega\sqrt{LC}$$

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\beta = \omega\sqrt{LC}$$

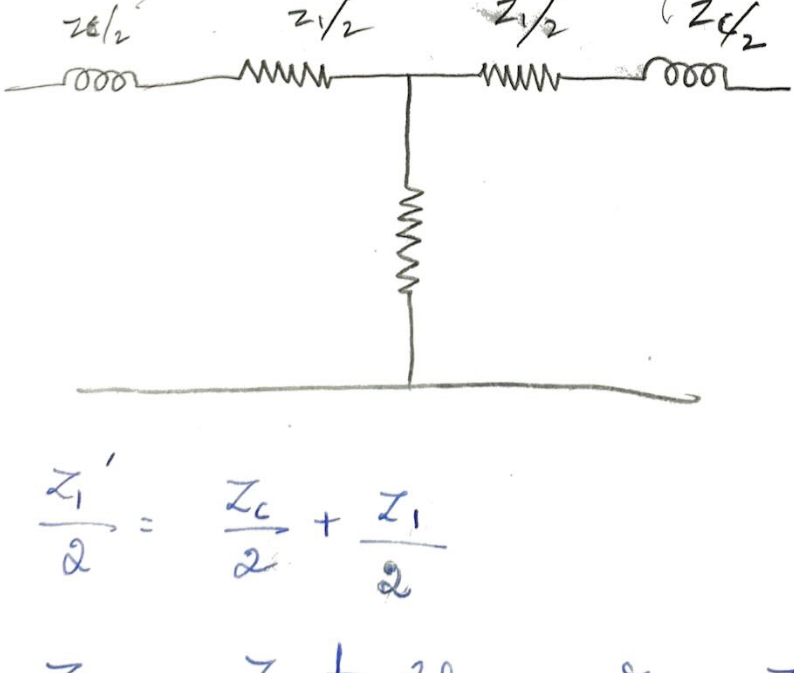
$$v = \omega/\beta = \frac{\omega}{\omega\sqrt{LC}} = 1/\sqrt{LC}$$

Since, $\alpha$ and $v$ are independent of frequency, the inductance loading makes the telephone cable as distortion less line (or) cable.

14.08.23

Campell's Equation

loaded coil.




$$\frac{Z_1'}{2} = \frac{Z_c}{2} + \frac{Z_1}{2}$$

$$\frac{Z_1}{2} = Z_0 \frac{\tan \gamma l}{2} \qquad Z_2 = \frac{Z_0}{\sin \gamma l}$$

$$\cosh \gamma l' = 1 + \frac{\frac{Z_1'}{2}}{Z_2}$$

$$= 1 + \frac{\left(\frac{Z_c}{2} + \frac{Z_1}{2}\right)}{Z_2}$$

$$= 1 + \frac{\left(\frac{Z_c}{2} + \frac{Z_0 \tan \gamma l}{2}\right)}{\frac{Z_0}{\sin \gamma l}}$$

$$= 1 + \frac{\frac{Z_c}{2}}{Z_0/\sinh \gamma l} + \frac{\frac{Z_0 \tanh \gamma l}{2}}{Z_0/\sinh \gamma l}$$

$$= 1 + \frac{Z_c \sinh \gamma l}{2Z_0} + \frac{\tanh \gamma l . \sinh \gamma l}{2}$$

$$= 1 + \frac{Z_c \sinh \gamma l}{2Z_0} + \frac{(\cosh \gamma l - 1)}{\sinh \gamma l} \times \sinh \gamma l.$$

$$= 1 + \frac{Z_c}{2Z_0} \sinh \gamma l + \cosh \gamma l - 1$$

$$\left[\frac{\tanh \gamma l}{2} = \frac{(\cosh \gamma l - 1)}{\sinh \gamma l}\right]$$

$$\cosh \gamma l' = \frac{Z_c}{2Z_0} \sinh \gamma l + \cosh \gamma l$$

This is known as Campell's equation

Inductance of a line not terminated by $Z_0$.

Waveform transforms from source to load $\rightarrow e^{\gamma s}$

Waveforms transfer from load to source then it is denoted as $e^{-\gamma s}$.

$$Z_s = Z_0 \left[ \frac{e^{\gamma s} + e^{-\gamma s}\left[\frac{Z_r - Z_0}{Z_r + Z_0}\right]}{e^{\gamma s} - e^{-\gamma s}\left[\frac{Z_r - Z_0}{Z_r + Z_0}\right]} \right]$$

(i) When the load impedance is terminated by characteristic impedance, then the input impedance is equal to characteristic ~~equation~~ impedance.

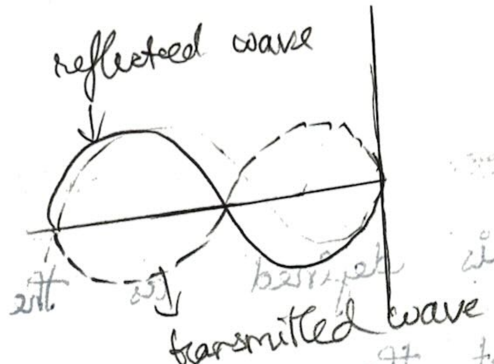
$$Z_R = Z_0$$

$$\boxed{Z_S = Z_0}$$

ii) When load impedance is terminated by characteristic impedance, the signals are absorbed in the load.

and signals are not reflected. Hence this line is called smooth line.

iii) When load impedance is not terminated by characteristic impedance, there is reflection and transmission wave. And this reflected and transmitted wave constitute a standing wave




Reflection coefficient:

It is defined as a ratio of reflected voltage to incident voltage at the receiving end of the line.

$$K = \frac{Z_r - Z_0}{Z_r + Z_0}$$

(i) $Z_r = Z_0$

$K = 0$ ; no reflection

(ii) $Z_r = 0$

$K = -1$ ; out of phase $\angle 180^\circ$

iii) $Z_r = \infty$

$$k = \frac{\frac{Z_r - Z_0}{Z_r}}{\frac{Z_r + Z_0}{Z_r}}$$

$\frac{1}{\infty} = 0$

$$= 1 - \frac{Z_0}{Z_r} \Big/ 1 + \frac{Z_0}{Z_r}$$

$$= 1 = \angle 0^\circ \text{ (in phase)}$$

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Return loss:

It is defined as the ratio of power at the receiving node to the power which is reflected at load.

$$\text{Return loss} = 20 \log \left| \frac{Z_R + Z_0}{Z_R - Z_0} \right|$$

Reflection loss

If $Z_R \neq Z_s$, reflection takes place. The power delivered to the load is less than the impedance matching, this reflection results in power loss. This loss is known as reflection loss.

reflection loss = $\ln\left[\dfrac{Z_R + Z_0}{2\sqrt{Z_0 Z_R}}\right]$ nephers or decibel (dB)

Reflection factor:

Reflection factor indicates the change in current in the load due to reflection at the mismatched junction (condition).

The ratio of current actually flowing in the load to the current that might flow under mismatched condition known as reflection factor.

$$\text{Reflection factor, } k = \frac{2\sqrt{Z_0 Z_R}}{(Z_R + Z_0)}$$

Input impedance & ~~Transfersion~~ Transfer impedance in transmission line - Open & Short circuited line

$$Z_S = Z_0 \frac{[Z_R \cosh \gamma l + Z_0 \sinh \gamma l]}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \rightarrow (1).$$

(i) Open circuited Condition

$Z_R = \infty$ ; $Z_S = Z_{OC}$

$$Z_{oc} = \frac{Z_0 [\cosh \gamma l + 0]}{[\sinh \gamma l]}$$

$$Z_{oc} = \frac{Z_0}{\tanh \gamma l}$$

Short circuited condition:

$$Z_R = 0 \quad ; \quad Z_s = Z_{sc}$$

$$Z_{sc} = \frac{Z_0 [0 + Z_0 \sinh \gamma l]}{Z_0 \cosh \gamma l + 0}$$

$$= \frac{Z_0 \cancel{\times} Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l}$$

$$Z_{sc} = Z_0 \tanh \gamma l$$

$$Z_{sc} \cdot Z_{oc} = Z_0 \tanh \gamma l \times \frac{Z_0}{\tanh \gamma l}$$

$$Z_{sc} Z_{oc} = Z_0^2$$

$$Z_0 = \sqrt{Z_{sc} Z_{oc}}$$

$$\frac{Z_{oc}}{Z_{sc}} = \frac{\frac{Z_0}{\tanh \gamma l}}{Z_0 \tanh \gamma l} = \frac{Z_0}{Z_0 \tan^2 h \gamma l}$$

$$\frac{Z_{oc}}{Z_{sc}} = \frac{1}{\tan^2 h \gamma l}$$

$$\tan^2 h \gamma l = \frac{Z_{sc}}{Z_{oc}}$$

$$\gamma l = \tan^{-2} h \left( \frac{Z_{sc}}{Z_{oc}} \right)$$